Worcester County Mathematics League

WOCOMAL Varsity Meet #2

Coaches' Booklet

December 3, 2003

December 3, 2003 **WOCOMAL Varsity Meet**

Round 1: Parallel Lines and Polygons

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. In trapezoid TGVR, base TG = x + 8 ; base RV = x + 2 ; and median SW = 2x + 3
   What is the sum of the lengths of the bases?

2. The angles of a convex polygon, P, have an average measure of 150 degrees.
   How many sides does polygon P have?

3. A polygon has 105 total sides and diagonals. How many sides does it have?

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Southbridge HS, Bromfield, and Mass Academy

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Round 2: Algebra 1 (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. The average of seven different **non-negative integers** is 7. What is the largest possible value of any of these numbers?

2. If a # b = a + ab + b, solve for **t** in terms of **c**: 2t # 7 = c # t

3. Two candles of equal length start burning at the same time. One will last 4 hours and the other 6 hours. How many hours will they burn before one candle is twice the length of the other?

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Burncoat, St. John's, and Doherty

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Round 3: Circles and Related Theorems

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. What is the area, in square units, of a square inscribed in a circle whose area is 25π square units?

2. Find the measure of the radius of an inscribed circle of a right triangle whose sides measure 5, 12, and 13.

3. A garden in the shape of a regular hexagon has an area of $150\sqrt{3}$ square feet. A walkway of uniform width surrounds the garden on all sides. In order for the garden and the walkway to contain the same area, the width of the walkway, written in simple radical form is: $a\sqrt{b} - a\sqrt{c}$ feet. What is the value of $(\mathbf{a + b + c})$?

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Tantasqua, Shrewsbury, and Assabet Valley

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Round 4: Sequences and Series (NO CALCULATORS)

ALL ANSWERS MUST BE AS DIRECTED IN THE PROBLEM.

1. Evaluate: $\sum_{n=1}^{8} (4 - 2n)$

2. What is the sum of the first two hundred multiples of 3 beginning with 3 ?

3. Determine the sum of the following infinite series as a reduced, improper fraction:

$$12 - 20 + 6 + 4 + 3 - 4/5 + 3/2 + 4/25 + \ldots$$

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Oxford, Quaboag, and Auburn

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Round 5: Matrices and Systems of Equations (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. Determine the value of $(x + y)$:

$$\begin{matrix} 3 & -1 \\ x & 2 \\ 4 & 8 \end{matrix} = 2 \begin{matrix} 3/2 & -1/2 \\ 5 & 1 \\ 2 & y+1 \end{matrix}$$

2. Determine the **sum** of the values of P if the following determinant has a value of zero.

$$\begin{vmatrix} -1 & 2 & 1 \\ 1 & 0 & P \\ -2 & P & 5 \end{vmatrix}$$

3. Solve the following system of equations over the set of **Real** numbers and write your solutions using "ordered pair" notation (x,y).

$$x^2 + y^2 + x + y = 8$$
$$xy + x + y = 5$$

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Westborough, Worcester Academy, Hudson

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TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON THE SEPARATE TEAM ANSWER SHEET. (2 points each)

1. Two chords in the same circle, neither of which are diameters, are perpendicular to each other. The point of intersection divides one chord into segments of 3 and 4 units and the other chord into segments that are 6 and 2 units. What is the diameter of the circle?

2. A circle of radius **r** is inscribed in a right triangle with hypotenuse **h**. What is the ratio of the area of the circle to the area of the right triangle?

3. Determine the y-intercept of the line passing through the points whose coordinates are (-3, 1) and (2, 5). Write your answer as a reduced, improper fraction.

4. The parallel sides of a trapezoid are 3 units and 9 units while the nonparallel sides are 4 units and 6 units. A segment is drawn parallel to the bases and that segment divides the original trapezoid into two smaller trapezoids having the same perimeter. What is the greatest ratio into which each of the nonparallel sides is divided?

5. The sum of the first **n** odd positive integers is 9409. What is the value of **n** ?

6. Write the following as an expression in the form $|\mathbf{Ax - B}| > \mathbf{C}$, where A,B, C are relatively prime integers: $\mathbf{x > 4}$ **or** $\mathbf{x < 1}$.

7. Determine the area, in square units, of a triangle whose vertices are:
(-3, 4), (1, -2), (and (4, 2).

8 Triangle PQR is a right triangle with angle R being the right angle and sides RP and RQ having lengths of 3 units and 8 units, respectively. Also, S is the point on the hypotenuse PQ such that RS bisects angle R. Find the length of the segment RS.

9. Find the sum of an arithmetic progression of 7 terms whose first term is 2 and whose 1^{st}, 3^{rd}, and 7^{th} terms form a geometric progression.

December 3, 2003 **WOCOMAL Varsity Meet ANSWERS**

Round 1: Parallel Lines and Polygon

1.	**(1 pt.)**	**14**
2.	**(2 pts.)**	**12**
3.	**(3 pts.)**	**15**

Round 2: Algebra 1-open

1.	**(1 pt.)**	**34**
2.	**(2 pts.)**	**(c-7)/(15-c)**
3.	**(3 pts.)**	**3**

Round 3: Circles & Theorems

1.	**(1 pt.)**	**50**
2.	**(2 pts.)**	**2**
3.	**(3 pts.)**	**14**

Round 4: Sequences and Series

1.	**(1 pt.)**	**-40**
2.	**(2 pts.)**	**60,300**
3.	**(3 pts.)**	**22/3**

Round 5: Polynomial Equations

1.	**(1 pt.)**	**13**
2.	**(2 pts.)**	**3**
3.	**(3 pts.)**	**(x,y) = (1,2)** **(x,y) = (2,1)**

TEAM ROUND (2 pts. Each)

1. $\sqrt{65}$

2. $\pi r/(r + h)$

3. 17/5

4. 4:1 or 4/1

5. 97

6. $|2x - 5| > 3$

7. 17

8. $(24/11)\sqrt{2}$

9. 35

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TEAM ROUND **School: _____**

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET. (2 points each)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

Total Points for Team Round: _____

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TEAM ROUND **School: _____**

Team #:_____

Team Members:

1. _____
2. _____
3. _____
4. _____
5. _____